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Firm boundaries and financing with opportunistic stakeholder behaviour

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Abstract

We explore the impact of strategic behaviour of equity holders, debt holders and an opportunistic supplier of a critical input on the firm's capital structure, organisational design, and its outsourcing decision. We show that the supplier can trigger strategic bankruptcy even when the firm is solvent. Equity holders respond to this either by eliminating the supplier and producing the input in-house or by reducing their exposure to debt by using equity-financing. Both responses introduce inefficiency since input costs are higher with in-house production, and debt is cheaper than equity. We show that the equilibrium debt-equity ratio varies positively with cash-flow profitability and the marginal cost of the supplier's input, but negatively with the riskiness of the cash flow and the equity holders' in-house input production costs.

Keywords: incomplete contracts, opportunistic behaviour, bankruptcy, capital structure. D0, C7, G3, L2

1. Introduction

A vast literature on capital structure examines the interactions between real and financial markets. A common theme in this literature is that a firm may use the composition of its financial claims as an instrument to deal with various imperfections arising from agency problems, informational asymmetries, regulation, non-competitive markets, and unions, among others.[1]

More recently, the stakeholder theory of capital structure has been discussed in several theoretical and empirical studies. The basic idea behind this theory is that a firm is a collection of its stakeholders (workers,

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[1] The earlier literature on this subject began with Jensen and Meckling (1976) and was developed by a series of papers in the 80s and 90s. See Harris and Raviv (1991) and Allen and Winton (1995) for an early summary of this literature. For a more recent survey, see Frank and Goyal (2008).

suppliers, creditors, customers, equity holders, etc.) and the relationships among them determine its financial and investment policies. For example, Titman (1984) argues that liquidation of a bankrupt firm hurts the long term prospects of its suppliers and customers due to "switching costs" (for example, the costs of locating new business partners. See Williamson (1975)).

These stakeholders take such costs into account when selling or buying from firms that run the risk of bankruptcy in the near future. Consequently, they reduce the volume of business with highly leveraged firms out of fear of bankruptcy. The anticipation of such behaviour by stakeholders in turn prompts firms to reduce their levels of debt. Following Titman (1984), the role of financing decisions in shaping relationships and bargaining power among stakeholders during bankruptcy has been extensively analysed theoretically and empirically.[2]

In recent times, stakeholder theory has been extended to previously unexplored areas.[3] A common feature of this literature is that each stakeholder acts independently. The possible formation of alliances by stakeholders' ex-post has not been explicitly recognised in the analysis of a firm's financing and investment decisions. In reality, however, alliances are often attractive as they may increase the ex-post power of stakeholders. This may in turn affect the ex-ante choices of financing and investment.

There are many real life examples that illustrate the power of stakeholder behaviour in the face of bankruptcy. One commonly cited example is the Chapter 11 bankruptcy filing of Olympia and York, where a breakaway group of creditors, suppliers and other initial stakeholders took over the former's Canary Wharf venture in London, and eventually drove out the original equity holders.[4]

We attempt to fill this gap by showing that the divergence of stakeholder interests becomes critical in the face of bankruptcy and this may affect a firm's ex-ante choice of capital structure and organisational

[2]The following represents a selection of the early literature in this area. Brander and Lewis (1986); Brander and Spencer (1989); Bronars and Deere (1991); Dasgupta and Sengupta (1993); Perotti and Spier (1993); Appelbaum (1993); Habib and Johnsen (1999); Hennessy and Livdan (2009); and Matsa (2010) provide theoretical analysis. On the other hand Titman and Wessels (1988); Kale and Shahrur (2007); and Banerjee et al. (2008), among others, provide empirical studies of the prevalence of low levels of debt. For a comprehensive survey of this literature, see Parsons and Titman (2008).

[3]The following is a selection of recent papers in this literature. Almazan et al. (2017) show how firms can use investment choices and the capital budgeting process to transmit information for the purpose of incentivising stakeholders (workers). Cen et al. (2015) analyse how decreased vulnerability as a result of anti-takeover provisions enhance firm value by helping firms build new customer relationships while preserving existing ones. Dasgupta et al. (2017) show how regulatory shocks affect product market competitiveness and may force greater CEO turnover in weakly governed firms, but improve performance and compensations in firms with better governance. Using cross-country data Ellul et al. (2018) show how publicly provided insurance leads to lower wages but greater employment stability, potentially leading to longer term stakeholder relationships. Lambrecht and Pawlina (2013) explain the emergence of negative net debt and debt conservatism in a set up where firms facing bankruptcy risk employ transferable human capital against which tradable financial claims cannot be issued. They show that a combination of such frictions together with a constraint on managerial finance may lead to negative net debt in equilibrium.

[4]Other instances of bankruptcy arising from conflict between stakeholders include the following: (i) The opportunistic multiple party bankruptcy bargaining, involving Wheeling-Pittsburgh Steel Corporation, its United Steelworkers of America workers, and its creditors. (ii) The case of Campeau where the suppliers triggered bankruptcy (nyt (1990)). (iii) The case of GM, where a debt-equity swap deal was initially rejected by a tough bargaining position by the creditors and the unions (wsj (2009)), (iv) the case of Skeena Cellulose Inc. where the creditors formed a joint venture with the British Columbia government as the majority stakeholder after severe financial woes. (v) Continental Airlines' use of bankruptcy threats to extract union concessions. (vi) Texaco's attempts to use insolvency to avoid payment of damages to its arch rival Pennzoil.

design. We analyse the choice of capital structure and organisational design made by the firm at the start through a non-cooperative lens, as it anticipates the possibility of opportunistic behaviour of the supplier in the case of bankruptcy.

We explore the interactions between three stakeholders, namely equity holders, debt holders and a supplier of a critical input.[5] We examine the co-determination of the choice of a firm's capital structure and its organisational design. In particular by capital structure we mean the debt-equity ratio, and by organisational design we mean the choice between procuring input from an outside supplier (outsourcing) and producing such input in-house. We analyse a previously undiscovered cost of the co-existence of debt holders and suppliers. This cost induces the firm to choose equity over debt and in-house production over outsourcing even when debt and outsourcing are cheaper.

In our set-up, the supplier provides a critical input to the production process so he is capable of running the firm with the debt holders. This ability to shut out the equity holders creates a potential for conflict when the firm is on the verge of financial distress and gives rise to a novel cost of debt. This creates a trade-off between cheap debt on the one hand and the possibility of strategic behaviour by the supplier (summed up in Proposition 2) leading to the co-determination of a firm's capital structure and its organisational design as described in the following paragraph.

If a firm issues more debt instead of equity to finance its capital requirements, it benefits as equity is more expensive than debt.[6] On the other hand, the cost of debt is the increased likelihood of strategic bankruptcy triggered by the supplier which results in the loss of surplus as the equity holders are shut out. The threat of strategic bankruptcy is dealt with in one of two ways – (a) firms with a low capital requirement respond by reducing the debt-equity ratio, whereas (b) firms with a large capital requirement shut out the outside supplier by producing the input in-house, and continue to rely on debt.

In our set-up, strategic bankruptcy is costly because it is a regime where the supplier pushes the equity and the debt holders to what their payoffs would be in case of non-strategic bankruptcy. Non-strategic bankruptcy is the standard bankruptcy regime where the equity holders receive nothing and the debt holders

[5] In this paper, we use the term outside "supplier" to represent a third party to the contract, who provides an input which is essential in the production process. To fix ideas, we consider this party the supplier. However, the model may extend to environments where the party may be a large investor, a partner in a venture capital firm, a skilled worker, or a complementary group of shareholders contributing value to the firm. The trade-off we highlight is between a superior technology that requires cooperation between multiple stakeholders vs. an inferior technology that can be operated jointly by a few stakeholders. This trade-off may arise in a more general model where the stakeholders are not specifically labelled. We are indebted to a referee for pointing out this out.

[6] In order to discuss a meaningful trade-off between debt and equity as a choice of financing, we assume that debt is cheaper that equity. This is micro founded on the idea that debt is subject to tax shields or explicit or implicit subsidies, like loan guarantees by the government. Equity may also be more expensive due to agency costs and informational asymmetry between the equity holders and the managers of the firm. Furthermore, equity being paid only in high states of nature is riskier due to informational asymmetries about the upside potential of the firm. With this, we attempt to analyse situations where firms issue equity even when it is more expensive than debt.

receive the liquidation value of the assets. As a result, in the state of bankruptcy, the equity holders withhold their input (effort) thereby reducing the value of the firm. The incremental surplus lost to shareholders in a strategic bankruptcy turns out to be the difference between the cash flow in solvent states (where both the equity holders and the supplier contribute inputs jointly) and in the state of bankruptcy when only the supplier provides an input. Essentially the (incremental) surplus lost is equal to the difference of firm value in case it is solvent and the same when the firm is a bankrupt entity. The optimal debt-equity ratio equates this cost of strategic bankruptcy to the benefit of cheap debt at the margin.

Based on this trade-off, we derive the optimal debt-equity ratio and show that this depends negatively on (a) the marginal cost of the supplier's input and (b) the riskiness of cash flow but depends positively on (c) the gross profitability of the project, and (d) the firm's own marginal cost of production of its own input. These variables affect the optimal debt-equity ratio because they impact either the probability of strategic bankruptcy or the lost incremental surplus in case of strategic bankruptcy.

When the state of nature is favourable, the supplier cannot trigger bankruptcy. On the other hand, no stakeholder can save the firm from going bankrupt when the state of nature is sufficiently unfavourable. This result is reminiscent of conclusions drawn by Hart and Moore (1998) where the equity holders cannot prevent a foreclosure because they lack of sufficient funds to compensate the supplier and the debt holders. However, for states of nature between these extremes, the supplier compares his own pay-off between solvency (when he supplies his input jointly with the equity holders and shares the pie with both the equity and debt holders) and bankruptcy where he provides the input and shares the proceeds with the debt holders alone. Since the supplier can only profitably trigger bankruptcy if he brings the debt holders on board, issuing debt ex-ante imposes the cost of potential strategic bankruptcy on the equity holders.

Our paper makes three contributions to the literature investigating the impact of stakeholders' complex relationship on a firm's choice of financing and organisational design. First, we find a novel cost of debt, hitherto unexplored in the literature, arising out of the supplier's incentives to withdraw critical input during economic distress. This cost leads to a choice of expensive equity financing over cheaper debt. Moreover this cost is relevant only for industries dependent on the supply of a critical input. Hence, a primary contribution of this paper is to provide explanations for why supplier dependent industries tend to use a smaller volume of debt in spite of its other benefits. Known as *"debt conservatism"* or *"zero leveraged firms"*, a large empirical literature establishes that firms eschew cheap debt and issue equity wherever supply considerations of crucial input play a big role.[7] While the existing explanations of the costs of debt including

[7]Titman and Wessels (1988), Qian (2003), Strebulaev (2007), Strebulaev and Yang (2013), Kale and Shahrur (2007), and Banerjee et al. (2008) among others, have examined the relationship between a firm and its primary stakeholders, such as suppliers and customers, and studied the effect of such relationships on the choice of capital structure. These empirical papers

"risk shifting", bankruptcy costs, or debt overhang, etc. hold true in general, the novelty of our result is to show the existence of a cost of debt that particularly arises when the suppliers can withdraw a critical input, thereby triggering both strategic bankruptcy and the destruction of firm value.

Second, we show that this possibility of strategic stakeholder behaviour affects not just the composition of financing but organisational design as it forces the firm to inefficiently expand its boundaries by eliminating the supplier and producing its critical input in-house. As a consequence, we derive new empirical predictions described as follows: Beyond a critical size, a firm will exclusively use cheaper debt but then produce its critical input in-house which could be obtained more cheaply outside. However below the critical size, a firm will outsource the production of its critical input but will be forced to use equity over cheaper debt. That is, debt conservatism, as found in the empirical literature would be more common in smaller or medium sized firms while larger firms will rely on debt but produce critical inputs in-house.

The hold-up problem in our model is not just due to the conflict between the equity holders and the supplier. Instead it stems from the inability of the equity holders to stop the suppliers from triggering bankruptcy. This source of friction has not been explored in prevailing literature.

Such hold-up problems create inefficiency as the firm is either forced to issue more expensive equity or produce the input in-house at a higher cost. This inefficiency is created by the inability to write a contract that specifies the terms of a future contract if a party to the contract defects (much in the same way that a first marriage contract cannot specify the terms of a potential second marriage contract if the first marriage breaks down and the parties re-marry). The basic premise of the paper is that while the equity holders can write a complete and enforceable debt contract, they cannot write such a contract with the suppliers. The two contracts are, therefore, asymmetric. It is the incompleteness of the contract with the suppliers that gives rise to opportunistic behaviour, namely, possible strategic bankruptcy.

The plan of the paper is as follows. Section 2 outlines the timeline of the model. In sections 3 and 4 we analyse the firm's capital structure and post-bankruptcy negotiations under outsourcing and in-house production, respectively. Section 5 presents the firm's choice between in-house production and outsourcing. We discuss our results and the role of key assumptions in our model in sections 5.1 and 5.2, respectively. Section 6 concludes.

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find that firms producing durable goods use specialised and non-substitutable inputs. Such firms that depend heavily on suppliers or supply chains for procurement of inputs tend to underuse debt as a source of external finance. Berk et al. (2010) show that empirically observed low debt ratio could emerge when a firm needs to compensate risk averse entrenched workers with higher compensation for insuring them from bankruptcy risk associated with leverage. Hence, a reduction in debt may reduce the wage premium. Lambrecht and Pawlina (2013) also explain debt avoidance in human capital intensive industries with a different mechanism (See footnote 3). Our setup differs from these papers as we introduce strategic bankruptcy which is not the focus of these papers.

2. Structure and timeline

We consider a firm that uses three inputs x, y, and K in its production process. The gross profits of the firm are

$$R(x, y, K, \theta) = \begin{cases} R_E(x) + R_S(y) + \theta & \text{if } y > 0 \text{ and } K \\ \theta & \text{if } y = 0 \text{ and } K \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where K denotes the capital required to start the enterprise, x is the input of the equity holder, and a random component θ captures the uncertainty facing the firm. We abuse notation slightly and use θ as both the random variable and its realisation. Input y is a critical input required for production, and it is either sourced from an outside supplier or produced in-house by the firm. Note that input y is critical since the absence of y implies that there is no additional return from investing x. The convex cost functions for x and y in the case of outsourcing are denoted by $C_E(x)$ and $C_S(y)$. We assume that $C_E'''(x) \geq 0$, and $C_S'''(y) \geq 0$. The key difference between outsourcing and in-house production is that the costs for x and y in case of in-house production are $\phi C_E(x)$ and $\phi C_S(y)$ where $\phi > 1$. That is, we assume that the costs of production are greater when the input is produced in-house. The assumption that in-house production raises the cost of x and y equally by ϕ simplifies the analysis as it ensures that the optimal mix of x and y remains the same with outsourcing and in-house production. However, our results remain unaffected if instead we assume that the cost of y increases by ϕ with in-house production whereas the cost of x remains unaffected.[8] Furthermore, the input of the equity holders and the supplier is non-verifiable and consequently, not contractible.

We assume that the random variable θ, is distributed over $[0, \infty)$ according to a density function $g(\theta)$ that is differentiable, and a corresponding cumulative distribution $G(\theta)$. The functions $R_E(x)$ and $R_S(y)$ are assumed to be differentiable, increasing and concave in x and y respectively, with $R_E(0) = R_S(0) = 0$. We assume that $R_E'''(x) \leq 0$, and $R_S'''(y) \leq 0$.[9] We also assume that $R_S(y)$ satisfies the Inada condition: $\lim_{y \to 0} R_S'(y) = \infty$. This is a sufficient (but not a necessary) condition that guarantees the optimality of giving the supplier incentives to provide input y. Finally, we assume that capital K fully depreciates at the end of the period.

[8]Although a vertically integrated firm could reduce the transaction costs of procuring input from outside, the process may also generate other costs due to losses in flexibility, and the sacrifice of gains from specialisation in stages of production. See Buzzell (1983), Perry (1989) and Grossman and Helpman (2002). Lafontaine and Slade (2007) present evidence that suggests that agency costs in the form of moral hazard may make vertical integration less attractive. Whether such costs dominate the gains from vertical integration is an empirical question. We analyse the interesting case where strategic behaviour at the stage of bankruptcy could force a firm to opt for vertical integration even when it is cheaper to procure the input from the market.

[9]The signs of the third derivatives of the revenue and the cost functions ensure that the desired second-order conditions in the equity holders' problem are satisfied.

We consider a multi-stage contracting game of symmetric information[10] with uncertainty. The game involves a principal (equity holders) and two agents: the debt holders and the supplier.

2.1. Timeline

The timeline, described in Figure 1, is as follows. At the start, the equity holders decide whether to purchase the specialised input from the supplier or produce it in-house. The upper segment of the timeline outlines events that unfold following the decision to buy the input from an outside supplier, which we describe as "outsourcing". The lower segment describes the events following the decision of the equity holders to produce the input in-house, which is akin to vertical integration. We do not analyse the question of whether the supplier is *"bought"* by the equity holders and made to produce the input in a vertically integrated set-up, or if the equity holders sets up an independent production unit for the input without relying on the supplier's machinery and skills. In our model, both interpretations are feasible and would lead to the supplier being eliminated as a strategic player from the game.

2.1.0.1. Stage 0: Choice of in-house production or outsourcing. In Stage 0, the equity holders choose between outsourcing and in-house production.

2.1.0.2. Stage 1: Contract with the debt holders. In Stage 1, the equity holders make their financing decision. In particular, we assume that the enterprise requires a fixed capital investment of K. The financing decision involves the choice of capital structure and is summarised by the contract with the debt holders. This contract specifies the level of debt, k_D, and the corresponding payment of D to the debt holders. Both these are endogenously derived. The amount of investment made through equity is then given by $k_E = K - k_D$. The capital market is assumed to be efficient, in the sense that there are no borrowing constraints.

2.1.0.3. Stage 2: Contract with the supplier. In case the equity holders choose outsourcing in Stage 0, in Stage 2 they sign a contract with the supplier. They sign an incomplete contract that specifies that the supplier will receive a share, $0 \leq \gamma(\theta) \leq 1$, of the surplus left over after the debt holders are paid, where the share itself may be state contingent and consequently depend on the realisation of θ. In addition to profit sharing, they also agree on a side payment, w, to be paid at the signing of the contract. The contract is therefore defined by the pair $\{\gamma(\theta), w\}$, and is endogenously determined. The contract, however, cannot specify precisely the amount of the inputs x and y as these are assumed to be non-contractible.

[10] The effects of asymmetric information on the firm's capital structure have been discussed in the literature extensively. See Jensen and Meckling (1976) and Leland and Pyle (1977).

2.1.0.4. Stage 3: Realisation of uncertainty. At the *"beginning"* of Stage 3, uncertainty about θ is resolved (and is common knowledge). Then, after the realisation of θ, in the case of outsourcing, the equity holders and the supplier simultaneously choose inputs x and y, respectively. The equity holders have to meet the legal obligations to the claimants: a payment of D to the debt holders and a share $\gamma(\theta)$ of the profits to the supplier. The outcome is constrained by the realised state of the world, the previously determined contracts and the existing legal structure. Specifically, the threat points in this bargaining game and consequently its outcome, are affected by bankruptcy laws and in particular by the seniority of claims.[11]

There are three possible outcomes in equilibrium. First, if the firm can meet all of its obligations, we have solvency. In this case, the overall game ends. On the other hand, if the firm's terminal assets, $R(x, y, K, \theta) \geq 0$, are insufficient to meet its obligations in full, the firm is in default and goes into bankruptcy.[12] But, there are two possible *"types"* of bankruptcy: strategic and non-strategic. Strategic bankruptcy occurs when an otherwise viable firm is forced into bankruptcy by the supplier. On the other hand, non-strategic bankruptcy occurs if the firm cannot meet its obligations due to an unfavourable state of the world (low θ), even when there is no opportunistic behaviour. In either case, if the firm goes into bankruptcy, its assets are distributed in accordance with the seniority of claims. We assume that the debt holders are secured creditors (thus first claimants), where their claim can be applied against $R_S(y) + R_E(x) + \theta$. The supplier is an unsecured creditor, and the equity holders are residual claimants.

In the case of in-house production, the equity holders choose both x and y. In this case, there is no question of the supplier strategically engineering bankruptcy. There is still the possibility of non-strategic bankruptcy when the revenues of the firm fall below the claims of the debt holders. Just as with outsourcing, we will see that this occurs when the realisation of θ is below an endogenously determined threshold.

2.1.0.5. Stage 4: Post bankruptcy bargaining. With outsourcing, if bankruptcy occurs in Stage 3, the debt holders and the supplier enter into post-bankruptcy bargaining. They decide on the level of the input y to be supplied and determine the division extra surplus that is generated. This stage is absent in the case of in-house production. A key assumption of our model when analysing the post-bankruptcy stage is that the equity holders cannot form an alliance with any of the stakeholders (the supplier or the debt holders) in the

[11] Bankruptcy laws in several jurisdictions have developed the "guidelines" that govern this bargaining process. The constraints imposed by the bankruptcy rules and the parties' relative strength in the bargaining process, determine the actual payoffs. For example, according to the bankruptcy laws in the US and Canada (See Altman (1983), Willes and Willes (2003), White (1980) and White (1989)), secured creditors receive the saleable value of the assets that are subject to security. If the value of the security is insufficient to satisfy the claims, the secured creditor is entitled to claim the remainder as an unsecured creditor. Unsecured assets are distributed, according to the US and Canadian laws in the following order: (i) administrative costs, (ii) taxes, (iii) wages and rents, (iv) unsecured creditors, and finally (v) equity holders. If several claimants have the same priority, they are paid on a pro-rata basis.

[12] See Titman (1984), White (1989), for examples of discussions of the corporate bankruptcy decision. See Aghion et al. (1992), for a discussion of efficient bankruptcy procedures. See also Hart and Moore (1998) for a model of default with renegotiations.

case of bankruptcy. We discuss this in section 5.2.

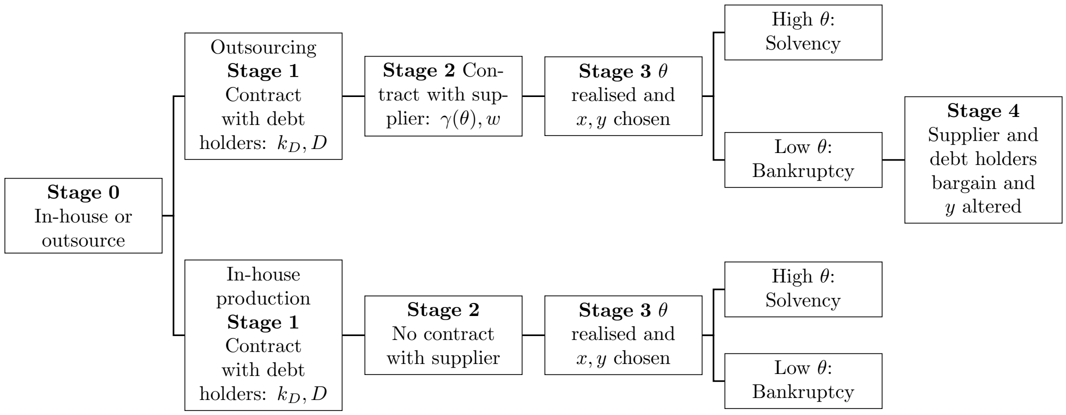

Figure 1: Timeline

We now examine the subgame perfect Nash Equilibrium of this game by solving the game backwards. In section 3, we analyse Stages 4 to 1 with outsourcing and in section 4 we do the same with in-house production. Finally, in section 5, we analyse whether the equity holders outsource y or choose in-house production.

3. Outsourcing: Stages 4 to 1

3.1. Stage 4: Post bankruptcy bargaining

If bankruptcy occurs in Stage 3 (the condition under which this happens will be discussed below), the debt holders and the supplier engage in post-bankruptcy bargaining in the last stage. Note that this stage only arises with outsourcing. In this bargaining game, they renegotiate the input y to be supplied. We assume that the supplier makes a take-it-or-leave-it offer to the debt holders.

With regards to the solution to this bargaining game, the threat payoff of the debt holders is θ since they can get this even when the supplier supplies no input. When y is supplied, the gain from trade is given by $R_S(y) - C_S(y)$. Since the joint surplus, $R_S(y) + \theta - C_S(y)$, is maximised at

$$R_S'(y^*) = C_S'(y^*), \tag{2}$$

it is clear that the bargaining solution must be such that $y = y^*$. The payoffs of the supplier, the debt holders, and the equity holders are respectively $p(\theta)$, $D(\theta)$ and $e(\theta)$ where

$$\begin{aligned} p(\theta) &= R_S(y^*) - C_S(y^*), \\ D(\theta) &= \theta, \\ e(\theta) &= 0. \end{aligned} \tag{3}$$

The existence of a region with strategic bankruptcy is robust to allowing the debt holders to have bargaining power at the post-bankruptcy stage. We show this in Appendix Appendix B by introducing a parameter that captures bargaining power. The current formulation is the special case when bargaining power is allocated entirely to the supplier which allows him to make a take-it-or-leave-it offer to the debt holders. As we explain in the appendix, this is done for analytical tractability. The equity holders' payoffs is 0 since bankruptcy has occurred. If bankruptcy *does not* occur, the game ends in Stage 3.

3.2. Stage 3: Realisation of uncertainty

The state of the world θ is revealed in the beginning of Stage 3. Thereafter the equity holders and the supplier simultaneously choose x and y respectively. Finally, the payoffs and the firm's solvency are determined. If the firm can meet its obligations, the game ends in Stage 3. If, on the other hand, the firm cannot meets its obligations, it goes into bankruptcy. Bankruptcy may or may not occur in this stage. If the realisation of the state is very poor, the firm enters bankruptcy and the game continues as outlined in the previous section. On the other hand, if both the supplier and equity holders expect solvency, then each chooses his own input to maximise his own payoff, given their respective shares γ and $1-\gamma$. In this case the equity holders maximise

$$\max_x (1-\gamma)(R_S(y) + R_E(x) + \theta - D) - C_E(x). \tag{4}$$

Similarly the supplier maximises:

$$\max_y \gamma(R_S(y) + R_E(x) + \theta - D) - C_S(y). \tag{5}$$

Let $x(\gamma)$ and $y(\gamma)$ be the solutions to the first order conditions such that

$$(1-\gamma)R'_E(x(\gamma)) = C'_E(x(\gamma)) \qquad \text{and} \qquad \gamma R'_S(y(\gamma)) = C'_S(y(\gamma)). \tag{6}$$

Unique solutions $x(\gamma)$ and $y(\gamma)$ are guaranteed to exist by concavity of the revenue functions and convexity of the costs.

Finally, note that bankruptcy cannot be triggered when $\theta > D$ since the debt holders receive θ in the post-bankruptcy negotiations, which cannot be greater than D, the amount they receive in case of solvency.[13]

We are now ready to characterise the possible equilibria in the sub games described in Stage 3 and 4.

$$\begin{aligned} \text{Define} \quad & v_S(x, y; \theta) := \gamma(R_S(y) + R_E(x) + \theta - D) - C_S(y), \\ \text{and} \quad & v_E(x, y; \theta) := (1 - \gamma)(R_S(y) + R_E(x) + \theta - D) - C_E(x) \end{aligned} \tag{7}$$

as the payoffs of the supplier and the equity holders in case of solvency with inputs x and y, and θ.

Assumption 1. *$v_S(x(\gamma), y(\gamma); \theta_H)$ is non decreasing in γ.*

This assumption ensures that the payoff of the supplier in the case of solvency is non-decreasing in the share of the profits γ allocated to him. As we show in the following lemma, θ_H is the threshold of θ over which the firm remains solvent. Note that production functions satisfying the Inada property of $\lim_{x \to 0} R'_E(x) = \infty$ are not consistent with Assumption 1. This is because if $R_E(x)$ satisfies the Inada condition there would exist a $\gamma < 1$ such that $v_S(x(\gamma), y(\gamma); \theta_H) > v_S(x(1), y(1); \theta_H)$.

Lemma 1. *With outsourcing there exists $\theta_L < \theta_H$ such that there is*

a) *non-strategic bankruptcy when $\theta \leq \theta_L$,*

b) *strategic bankruptcy when $\theta_L < \theta < \theta_H$, and*

c) *solvency when $\theta_H \leq \theta$.*

Proof. All proofs can be found in Appendix Appendix A. □

Figure 2 illustrates the result in Lemma 1. The lemma above partitions the production shock θ into three regions. When $\theta \geq \theta_H$, the firm has sufficient funds even in the absence of input by the supplier. As a result, there is no bankruptcy. Since the supplier can ensure that $R_E(x) = R_S(y) = 0$ by withholding y, we have $\theta_H = D$. That is, when $\theta \geq D$, the firm can meet its debt obligation even in the absence of the supplier's input. Similarly, when $\theta \leq \theta_L$ the firm's revenue is low enough such that the payoff of either the supplier or the equity holders is negative from supplying $y(\gamma)$ or $x(\gamma)$ once the debt holders are paid off. That is $\theta_L := \max\{\theta\}$ such that ;

$$\begin{aligned} \text{either} \quad & v_S(x(\gamma), y(\gamma); \theta) \leq 0, \\ \text{or} \quad & v_S(0, y(\gamma); \theta) \leq 0 \qquad \text{and} \quad v_E(x(\gamma), y(\gamma); \theta) \leq 0. \end{aligned} \tag{8}$$

[13] The supplier does not threaten to withhold input y if $\theta > D$ since such a threat is not be credible. The supplier would have payoff of zero if he were to carry out that threat, as the equity holders would pay D to the debt holders and keep the remaining $\theta - D$. The supplier is better off providing y and receiving $v_S > 0$.

We interpret this as a region where there is bankruptcy even without strategic behaviour by the supplier. The novel region is $\theta_L < \theta < \theta_H$ when the supplier's input is pivotal for ensuring solvency of the firm. In this region, the supplier triggers bankruptcy to secure a higher payoff even though he receives a positive payoff by ensuring solvency. In the following sections we describe each of these three possibilities.

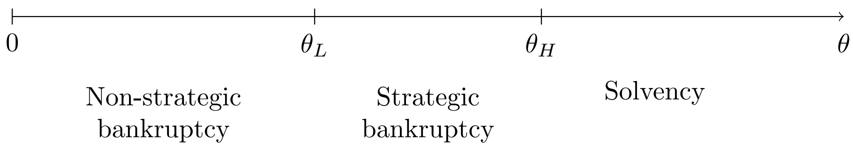

Figure 2: Outcomes for different values of θ with outsourcing

The equity holders choose their effort after the realisation of θ. Since they are aware that the supplier withholds y when $\theta \leq \theta_H$, it is an equilibrium for the equity holders in this region to choose $x = 0$. This is because of the assumption that the supplier's input is critical and therefore $x = 0$ is the best response when y is withheld for $\theta \leq \theta_H$. Although we focus on this equilibrium, we note that it may not be unique. It is possible that equity holders may incentivise the supplier to not trigger strategic bankruptcy by choosing a high $x > x(\gamma)$. This would make it profitable for the supplier to exert $y(\gamma)$ as his payoff from complying with the original contract would now be higher. In such an equilibrium, the interval of θ with strategic bankruptcy would be smaller but would continue to exist since the equity holders have to ensure a payment of D to the debt holders and a payment of $R_S(1) - C_S(1)$ to the supplier. This is not feasible while maintaining zero profit for the equity holders for low realisations of θ.[14]

3.2.1. Solvency

The solvency region is when $\theta_H \leq \theta$. If θ is large enough, the firm can always meet all its obligations to claimants, regardless of how much of y is actually supplied. If the supplier withholds y he gets the contracted share $v_S(x(\gamma), 0; \theta) = \gamma(\theta - D)$. But, by supplying $y = y(\gamma)$, he receives an additional $\gamma R_S(y(\gamma)) - C_S(y(\gamma))$. Hence $y(\gamma)$ is chosen by the supplier as it is the optimal input for the supplier when he receives a share γ. The payoff of the supplier, the debt holders, and the equity holders respectively is

$$\begin{aligned} p(\theta) &= v_S(x(\gamma), y(\gamma); \theta), \\ D(\theta) &= D, \\ e(\theta) &= v_E(x(\gamma), y(\gamma); \theta). \end{aligned} \tag{9}$$

[14] We thank an anonymous referee for pointing this out.

3.2.2. Strategic bankruptcy

Strategic bankruptcy occurs when $\theta_L < \theta < \theta_H$. Within this region, the supplier can *force* the firm into bankruptcy by not supplying his input, since this triggers a missing payment of $D - \theta > 0$ to the debt holders. To determine whether it is indeed in the interest of the supplier to exercise his option to force default, we compare his payoffs if he forces bankruptcy to when he does not. We know that if he forces bankruptcy, he will move to post-bankruptcy bargaining with debt holders in Stage 4. In case of default, at the end of post-bankruptcy bargaining he receives $R_S(y(1)) - C_S(y(1))$, since the optimal effort choice in case of strategic bankruptcy derived earlier is $y^* = y(1)$. Due to Assumption 1, when $\theta < \theta_H$ we have $R_S(y(1)) - C_S(y(1)) > v_S(x(\gamma), y(\gamma); \theta)$ and consequently when $\theta < \theta_H$, the supplier is always better off *actually* forcing bankruptcy.

Thus, in this region the supplier first forces bankruptcy by first withholding y, and later in Stage 4 choose $y = y(1)$. The equity holders, anticipating that the supplier will force bankruptcy, supply $x = 0$ since the supplier's input is critical in production. The players' payoffs are the same as the ones described in (3). This case implies that, under some circumstances due to opportunistic behaviour, the firm will not be able to prevent bankruptcy, even if it is viable (in the sense that the firm can survive if $y(\gamma)$ is supplied).

3.2.3. Non-strategic bankruptcy

When $\theta \leq \theta_L$ the firm will always default on its obligations to the debt holders, so that the equity holders are out of the picture and we have $x = 0$. The supplier withholds y and the game moves to Stage 4, post-bankruptcy bargaining. Assuming that the sharing rule does not depend on whether the firm went bankrupt, or was forced into bankruptcy, the gains from trade and the outcome of bargaining will be the same as the strategic bankruptcy case. That is $y = y(1)$ and the payoff of parties is the one described in (3).

3.3. Stage 2: Contract with the supplier

In Stage 2, after the contract with the debt holders is signed in Stage 1, the equity holders sign a contract with the supplier if outsourcing is used. This stage is absent with in-house production. The contract with the supplier specifies that he will provide his services and, in return, he will receive a share $\gamma(\theta)$ of the surplus left over after the debt holders are paid and a side payment of w will be paid at the signing of the contract. The input service, however, is non-verifiable so that the parties cannot sign a contract that is based on the level of y. Rather than restricting the equity holders to only offering a constant share γ to the supplier, we allow them to offer a share $\gamma(\theta)$ that is conditional θ since x and y are chosen after θ is realised.

Consider the firm's problem. Given the contract with the debt holders, the outcomes of the multilateral bargaining game and the supplier's choice of $y(\gamma)$, the equity holders' receipt in the last stage will be

$$e(\theta) = \begin{cases} v_E(x(\gamma(\theta)), y(\gamma(\theta)); \theta) & \text{if solvency, and} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

The equity holders choose a sharing arrangement, that is the function $\gamma(\theta)$, and the fixed payment w that maximises

$$\mathbb{E}(e(\theta)) - w - \rho_E k_E \tag{11}$$

where $\rho_E k_E$ is the capital requirement that is funded through equity decided in Stage 1 (see section 3.4) subject to the participation constraint of the supplier given by

$$\begin{aligned} \mathbb{E}(p(\theta)) + w &\geq 0 \\ \Leftrightarrow \quad G(\theta_H)(R_S(y(1)) - C_S(y(1))) + \textstyle\int_{\theta_H}^{\infty} v_S(x(\gamma(\theta)), y(\gamma(\theta)); \theta) g(\theta) d\theta + w &\geq 0. \end{aligned} \tag{12}$$

The left hand side of the inequality in (12) is the pay-off of the supplier which is an expectation over bankruptcy and solvency. As shown in (3), the first term is the payoff in the state of bankruptcy multiplied by its probability. The second term is the conditional expectation of the supplier's payoff in case of solvency, as seen in (7).

Note that this is a functional analysis problem since the equity holders choose a function $\gamma(\theta)$ and a constant w that maximises their payoff. Since w can be used to extract all surplus from the supplier, (12) must bind with equality and the maximisation problem can be analysed in two parts: First, the problem of choosing $\gamma(\theta)$ that maximises the total surplus, and second, the problem of choosing w such that the participation constraint of the supplier binds.

w is the constant component of the payment made to the supplier by the equity holders. This component is paid at Stage 2 when the contract with the supplier is signed. In equilibrium, w is always negative. This implies that at Stage 2 the supplier makes a flat payment to the equity holders. This is because the contract with the supplier is designed to give him incentives to supply y which is otherwise non-contractible. We assume that the supplier is not financially constrained and can pay $-w$, and is willing to do so as long as he recovers this in expectation (the inequality in (12)). The other component of the supplier's payoff is $\gamma(\theta)$, the share of profit, that will be made in Stage 3 conditional on solvency, and the payoff the supplier receives in the event of bankruptcy in Stage 4 when he negotiates with the debt holders. Taking expectations over this component, the equity holders set w to push the supplier to his outside option of zero at the ex-ante stage (Stage 2). We are now ready to characterise the choice of $\gamma(\theta)$ by the equity holders.

Proposition 1. *With outsourcing, the equity holders' agreement with the supplier is characterised as follows.*

a) *A unique constant γ^* exists that is chosen by the equity holders.*

b) *The interior solution for γ^* satisfies*

$$\frac{1-\gamma^*}{\gamma^*} = -\frac{R_E'(x(\gamma^*))}{R_S'(y(\gamma^*))} \cdot \frac{x'(\gamma^*)}{y'(\gamma^*)} \tag{13}$$

and this is increasing in the return to the supplier's input, relative to the equity holders' input.

This proposition establishes that the agreement between the equity holders and the supplier takes the form of $\gamma(\theta)$ being a constant. This result arises from the additive separability of the firm's revenue in θ. The proposition shows that a unique γ exists that maximises the payoff of the equity holders. As expected, this is increasing in the returns from the input of the supplier – it is in the interest of the equity holders to increase the supplier's share as the supplier's input is non-contractible and a greater γ induces a greater input. Since the choice of inputs in case of solvency does not depend on the level of θ, $\gamma(\theta)$ is constant. The equity holders will take γ^* as given when they make their financing decision.

3.4. Stage 1: Contract with the debt holders

In Stage 1, the equity holders sign a contract with the debt holders. The contract specifies the level debt k_D and the corresponding payment D. Note once again that, for an exogenously given K, the choice of k_D also determines the amount of capital funded through equity since $k_D + k_E = K$. The cost of equity financing is therefore $\rho_E(K - k_D)$ where $\rho_E > 0$ represents the parameter capturing the cost of equity. Similarly the interest rate available to the debt holders if they don't invest in the enterprise is $\rho_D > 0$. We assume that $\rho_E \geq \rho_D$.[15] More generally $\rho_D k_D$ is the minimum principal and interest on debt k_D.

The debt holders' participation constraint is

$$\begin{aligned} \rho_D k_D &\leq \mathbb{E}(D(\theta)) \\ &\leq \textstyle\int_0^D \theta g(\theta) d(\theta) + (1 - G(D))D. \end{aligned} \tag{14}$$

This participation constraint defines the market debt supply function.

Given the outcomes in Stages 2; 3; and 4, and noting that $\theta_H := D$,we find that the equity holders'

[15] All our results for the case $\rho_E = \rho_D$ also apply to the case when equity is cheaper than debt, that is, $\rho_E < \rho_D$.

receipts are given by

$$\begin{aligned} & \mathbb{E}(e(\theta)) - w - \rho_E(K - k_D) \\ = \; & \textstyle\int_D^\infty (R_S(y(\gamma^*)) + R_E(x(\gamma^*)) + \theta - D - C_S(y(\gamma^*)) - C_E(x(\gamma^*)))g(\theta)d\theta \\ & + G(D)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D). \end{aligned} \tag{15}$$

The equity holders choose a contract with the debt holders to maximise the expected net present value of their receipts (15), subject to the debt holders' participation constraint in (14), and the supplier's incentive constraint in (6). Note that since we used the supplier's participation constraint from (12) in the derivation of (15), its satisfaction has already been imposed. Furthermore, the optimal contract with the supplier, that is, $\gamma = \gamma^*, w = -\mathbb{E}(p(\theta))$, has also already been taken into account in the derivation of (15). The equity holders' problem is therefore to maximise (15) with respect to k_D and D subject to the participation constraint of the debt holders in (14).

Again, it is clear that the optimal solution does not leave extra surplus for the debt holders. In other words, their participation constraint will hold with strict equality. That is

$$\mathbb{E}(D(\theta)) = \rho_D k_D \tag{16}$$

If we substitute this strict participation constraint into (15) by substituting for k_D we get expected net present value of equity which the equity holders maximise;

$$\max_D \mathbb{E}(e(\theta)) + \mathbb{E}(p(\theta)) - \rho_E \left(K - \frac{\mathbb{E}(D(\theta))}{\rho_D} \right). \tag{17}$$

This is the expected net present value of the firm. The first term $\mathbb{E}(e(\theta))$ represents the net payoff of the equity holders. This is the equity holders' share of revenue net of the payment to the debt holders in case of solvency, and net of the cost of $x(\gamma)$. Similarly, the second term $\mathbb{E}(p(\theta))$ is the supplier's net expected payoff. The last term is the cost of equity financing ρ_E times the amount of capital financed through equity. When the entire capital requirement K is financed through debt, the last term equals zero as $k_D = K$, and the payments to the debt holders are subtracted from $\mathbb{E}(e(\theta))$ and $\mathbb{E}(p(\theta))$. On the other hand, when capital is fully financed through equity then $\mathbb{E}(D(\theta)) = 0$ and consequently $\mathbb{E}(e(\theta))$ and $\mathbb{E}(p(\theta))$ are correspondingly larger as no payment is made to the debt holders. Since the equity holders push both the debt holders and the supplier to their outside options in Stage 1 and 2 respectively, the expression in (17) represents the net ex-ante surplus generated by the firm.

Proposition 2. *With outsourcing,*

a) *when $\rho_E > \rho_D$ and the objective function is concave, there is a unique optimal $k_D^* \in [0, K]$.*

b) *when $\rho_E = \rho_D$, the probability of an opportunistic bankruptcy is zero and the optimal capital structure is $k_D^* = 0$ and all capital is financed through equity.*

The proposition above gives us an important property of the optimal capital structure, that there is no debt financing and consequently no strategic bankruptcy in equilibrium, when debt is as expensive as equity. In this case, there is no strategic bankruptcy as there is no debt to trigger default. The proposition also characterises what the optimal capital structure for the firm is. Define

$$\Delta TS := R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - (R_S(y(1)) - C_S(y(1))) \tag{18}$$

as the increase in the net surplus generated from the equilibrium inputs from the supplier and equity holders as we move from bankruptcy to solvency. When the solution to (17) is interior, D^* is characterised by the first order condition;

$$(\rho_E - \rho_D)(1 - G(D^*)) - \Delta TS g(D^*)\rho_D = 0. \tag{19}$$

The second-order condition that guarantees concavity is derived in the proof of the proposition in the appendix. Since the participation constraint of the debt holders binds, there is a positive monotonic relationship between k_D^* and D^*. In particular, from equation (16) we have

$$\rho_D k_D^* = \int_0^{D^*} \theta g(\theta) d\theta + D^*(1 - G(D^*)) \qquad \text{and} \qquad \frac{\partial k_D^*}{\partial D^*} = \frac{1 - G(D^*)}{\rho_D} > 0. \tag{20}$$

Therefore the properties of D^* that we derive also apply to k_D^* which is the level of debt financing.

Equation (19) shows the trade-off behind the optimal choice of financial structure. The left hand side is the gain resulting from cheaper debt financing times the probability that the firm is solvent. The cost of debt financing is the loss of the extra surplus due to bankruptcy times the increase in the probability of strategic bankruptcy that such extra debt induces. As such, this is the key trade-off between choosing to finance the firm's capital requirement through debt vs. equity. First, note that an interior solution obtains only if $\rho_E > \rho_D$. When debt is cheaper than equity, on the one hand by marginally increasing D, and therefore k_D, which is the capital financed through debt, the firm makes a lower payment relative to equity financing. This is captured in the first term $(\rho_E - \rho_D)(1 - G(D^*))$. On the other hand, increasing D and therefore k_D introduces the possibility of costly strategic bankruptcy. In particular, strategic bankruptcy leads to zero input by the equity holders and consequently a loss of surplus of ΔTS. D^* is the value of D that balances this trade-off. Hence optimal debt is determined by the trade-off between cheaper debt on the one hand and the surplus lost due to strategic bankruptcy on the other. This is illustrated in the example below.

3.4.0.1. Example. If θ is uniformly distributed on the interval $[0, \overline{\theta}]$, we can rewrite the first order condition from (19) as

$$(\rho_E - \rho_D)\left(1 - \tfrac{D^*}{\overline{\theta}}\right) = \rho_D \tfrac{\Delta TS}{\overline{\theta}} \tag{21}$$

$$\Leftrightarrow \qquad D^* = \overline{\theta} - \tfrac{\rho_D}{\rho_E - \rho_D}\Delta TS. \tag{22}$$

We observe that the optimal debt depends negatively on the cost of debt ρ_D, positively on the cost of equity ρ_E, and negatively on ΔTS; the incremental surplus lost in strategic bankruptcy.

ΔTS itself depends on basic parameters like costs of inputs provided by the equity holders and the supplier. Moreover, with a general distribution of θ, the properties of the distribution such as its mean and variance will also affect the optimal level of debt. The proposition below derives the impact of these underlying parameters on the equilibrium debt in the form of comparative statics that form the basis of the testable implications of this paper.

Proposition 3. *Assume the interior solution to D^* obtains.*

a) *Let distribution of θ improve from $G(\cdot)$ to $F(\cdot)$ with the corresponding densities $g(\cdot)$ and $f(\cdot)$, with $F(\cdot)$ dominating $G(\cdot)$ in the sense of the monotone hazard-rate condition (MHRC)*

$$\frac{f(\theta)}{1 - F(\theta)} \leq \frac{g(\theta)}{1 - G(\theta)} \qquad \forall \theta. \tag{23}$$

Then

$$D^*(F(\cdot)) \geq D^*(G(\cdot)) \tag{24}$$

b) *When the marginal cost of inputs x and y is constant at c_E and c_S respectively, then*

$$\frac{\partial D^*}{\partial c_E} \geq 0 \qquad \text{and} \qquad \frac{\partial D^*}{\partial c_S} \leq 0. \tag{25}$$

This proposition shows some comparative statics results that shed light on how the capital structure of the firm varies with the environment. Part a) of the proposition shows that as the distribution of θ improves in the sense of the MHRC, the debt-equity ratio increases.[16]

Note that MHRC implies first order stochastic dominance. This in turn implies second order stochastic dominance. Hence if $F(\cdot)$ dominates $G(\cdot)$ in the MHRC sense, then it must also dominate $G(\cdot)$ in the first and

[16] Since total capital requirement K is constant, an increase in D implies an increase in k_D and an increase in the debt-equity ratio $\frac{k_D}{K - k_D}$.

second order stochastic dominance sense.[17] A decrease in the hazard rate $\frac{f(\theta)}{1-F(\theta)}$ implies that the realisation of the random component of the cash flow is more likely to be inclined towards the better states, and this reduces the probability and expected costs associated with strategic bankruptcy. Since the hazard rate bears a negative relationship with the riskiness of cash flow, our model predicts an inverse relationship between the riskiness of cash flow and optimal debt-equity ratio.

Part b) of the proposition shows how the capital structure of the firm changes in the cost of inputs for the equity holders and the supplier. We see that D^* and therefore k_D^*, which is the level of capital financed through debt, is increasing in the marginal cost of the equity holders' input and decreasing in the marginal cost of the supplier's input. As the equity holders' input becomes more expensive, it becomes more attractive to rely on the supplier's input. This reduces the cost of strategic bankruptcy which is a state where only the supplier makes an input as the equity holders are shut out. Therefore D^* increases; increasing the likelihood of strategic bankruptcy. Conversely, as the input of the supplier becomes costlier, it becomes more attractive to rely on the input of the equity holders, and the state of strategic bankruptcy where only the supplier provides an input, becomes less attractive. Therefore D^* decreases; reducing the likelihood of strategic bankruptcy.

Finally, it is worth summarising the general predictions and empirical implications of the model. Our model identifies a new cost of debt financing. This new cost is due to two factors:

1. The probability of opportunistic behaviour leading to strategic bankruptcy $G(D^*) - G(\theta_L)$.
2. The firm cannot control the potential post-bankruptcy arrangement between the supplier and the debt holders.

Due to this new cost, the firm faces a new consideration that tends to limit its use of debt. Since, in general, there are other motives for using debt, our conclusion simply implies that under these conditions, the debt-equity ratio will tend to be lower. At the same time, it is important to remember that it is the incompleteness of the contract with the supplier that facilitates opportunistic behaviour in the first place.

4. In-house production Stages 3 to 1

Note that there is no Stage 4 and Stage 2 with in-house production as the supplier is absent (see Figure 1). Hence we can focus on Stage 3 and Stage 1.

[17]See Proposition 4.3 in Wolfstetter (2002).

4.1. Stage 3: Realisation of uncertainty

Once θ is realised, as there is no supplier, the equity holders' problem is

$$\max_{x,y} R_E(x) + R_S(y) + \theta - D - \phi C_E(x) - \phi C_S(y). \tag{26}$$

The positive optimal in-house (i) inputs x_i, y_i as the values of x and y that satisfy the first order conditions

$$R'_E(x_i) = \phi C'_E(x_i) \qquad \text{and} \qquad R'_S(y_i) = \phi C'_S(y_i). \tag{27}$$

Hence the solution to the equity holders' problem is to choose

$$x^*, y^* = \begin{cases} x_i, y_i & \text{if } R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i) \geq 0 \\ 0, 0 & \text{otherwise.} \end{cases} \tag{28}$$

With this we can characterise the outcomes in Stage 3 with in-house production.

Lemma 2. *With in-house production there exists a θ_B such that there is*

a) *non-strategic bankruptcy when $\theta < \theta_B$, and*

b) *solvency when $\theta_B \leq \theta$.*

Figure 3 illustrates the result in Lemma 2. The characterisation with in-house production is similar to that of outsourcing with the exception that there is no supplier to engineer bankruptcy. With in-house production, there are two possibilities that we discuss below.

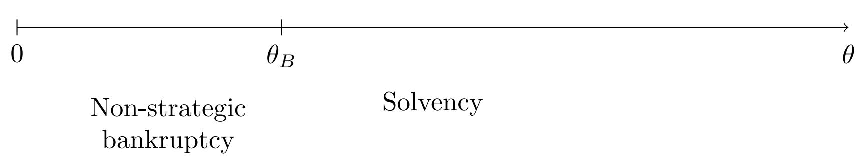

Figure 3: Outcomes for different values of θ with in-house production

Since θ_B is defined as the non-strategic bankruptcy threshold with in-house production we have

$$\theta_B = \max\{0, D + \phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i)\}. \tag{29}$$

It is easy to see that θ_B is lower than θ_H, the strategic bankruptcy threshold with outsourcing since $\theta_H = D$ and $\phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i) < 0$. The comparison between θ_B and θ_L, the non-strategic bankruptcy threshold with outsourcing, is less obvious. If outsourcing generates more surplus relative to in-house production, then $\theta_L < \theta_B$. Intuitively, this is because for the same realisation of θ, outsourcing generates more profit than in-house production making the threshold for bankruptcy lower with outsourcing. This occurs if inequality in (36) is satisfied as discussed below. On the other hand, if in-house production generates a larger surplus, we will have $\theta_L > \theta_B$.

4.1.1. Solvency

If the realisation of θ is high enough (greater than θ_B), such that the revenues of the firm are sufficient to meet the debt holders' claim of D, the firm remains solvent. In this case the payoffs of the debt and equity holders are

$$\begin{aligned} D(\theta) &= D \\ e(\theta) &= R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i), \end{aligned} \tag{30}$$

respectively.

4.1.2. Non-strategic bankruptcy

If the realisation of θ is low enough (less than θ_B), such that the revenues of the firm are insufficient to meet the debt holders' claim of D, the firm goes bankrupt. In this case, the payoffs of the debt and equity holders are

$$\begin{aligned} D(\theta) &= \theta \\ e(\theta) &= 0, \end{aligned} \tag{31}$$

respectively.

4.2. Stage 1: Contract with the debt holders

Given the outcomes in Stages 2; 3; and 4, and noting that $\theta_B := D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)$, the equity holders' receipts are given by

$$\begin{aligned} & \mathbb{E}(e(\theta)) - \rho_E(K - k_D) \\ = \; & \textstyle\int_{\theta_B}^{\infty} (R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + \theta - D) g(\theta) d\theta - \rho_E(K - k_D). \end{aligned} \tag{32}$$

Again, it is clear that the optimal solution does not leave extra surplus for the debt holders. In other words, their participation constraint will hold with strict equality. That is,

$$\mathbb{E}(D(\theta)) = \int_0^{\theta_B} \theta g(\theta) d\theta + (1 - G(\theta_B))D = \rho_D k_D. \tag{33}$$

If we substitute this strict participation constraint into (32) by substituting for k_D we find that the expected net present value of equity that the equity holders maximise is

$$\max_{D} \mathbb{E}(e(\theta)) - \rho_E \left(K - \frac{\mathbb{E}(D(\theta))}{\rho_D} \right). \tag{34}$$

The first term $\mathbb{E}(e(\theta))$ represents the net payoff of the equity holders in the case of in-house production. It is composed of the revenue of the firm net of the payment to the debt holders in the case of solvency, and the cost of x_i and y_i. The last term is the cost of equity financing ρ_E times the amount of capital financed through equity. Higher debt financing leads to larger payment to the debt holders reducing $\mathbb{E}(e(\theta))$ whereas higher equity financing increases $\mathbb{E}(e(\theta))$ and reduces $\mathbb{E}(D(\theta))$. Comparing this to the analogous expression for the case of outsourcing presented in (17), we see that the payment to the supplier is absent since now the input y is produced in-house. We can now characterise the choice of financing with in-house production.

Proposition 4. *With in-house production,*

a) *when $\rho_E > \rho_D$ and the objective function is concave, there is a unique optimal $k_D^* \in [0, K]$.*

b) *only equity financing is chosen when $\rho_E = \rho_D$.*

The second-order condition that guarantees concavity is derived in the proof of the proposition in the appendix. We observe that even with in-house production, the choice of financing doesn't just depend on whether $\rho_E > \rho_D$. This is because even with in-house production there are realisations of θ, in particular when $\theta < \theta_B$, when sub-optimal $x = y = 0$ is chosen. This is reminiscent of Myers (1977) where under-investment occurs when the equity holders cannot renegotiate the existing debt. This would change if we instead assume that the equity holders can renegotiate with the debt holders in a way that the equity holders can retain the net returns from their input x and y namely $R_E(x) + R_S(y) - \phi C_E(x) - \phi C_S(y)$. With this modification, the efficient level of $x = x_i$ and $y = y_i$ will always be provided. Moreover, capital will either be financed only through debt, if $\rho_E > \rho_D$, or only through equity, if $\rho_E = \rho_D$. All other results will remain qualitatively unchanged. We rule out such renegotiation between the equity and the debt holders to maintain consistency: since this is what we have also assumed about outsourcing.

5. Stage 0: In-house production or outsourcing

In this section, the equity holders choose between in-house production and outsourcing of the critical input y. First, we show how equity and debt financing varies with their costs.

Proposition 5. *With both outsourcing and in-house production, the debt-equity ratio is weakly increasing in $\frac{\rho_E}{\rho_D}$, the relative cost of financing through equity.*

This shows that as the relative cost of equity increases, the amount of debt financing relative to equity financing increases. Next, we analyse how the capital structure varies with $\frac{\rho_E}{\rho_D}$ with outsourcing and in-house production. To ease the analysis we assume:

Assumption 2.

$$R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) \geq \rho_D K.$$

This implies that with in-house production, the firm remains solvent regardless of the realisation of θ. Note however that strategic bankruptcy with outsourcing continues to remain possible: since this occurs whenever $\theta < D$. We can now characterise the equity holders' decision of outsourcing and in-house production. To begin with note that when

$$R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) > R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) \tag{35}$$

in-house production will always be preferred over outsourcing since the payoff from outsourcing even when the firm is solvent, is weakly lower than in-house production. Similarly if

$$R_S(y(1)) - C_S(y(1)) > R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i), \tag{36}$$

then outsourcing will be preferred over the in-house production of y since the payoff from outsourcing is greater even in the event of bankruptcy. The interesting parameter space is in the intermediate range when the payoff from outsourcing, relative to in-house production, is greater in the event of solvency, but lower in case of bankruptcy. Focusing on this region allows us to highlight the trade-off between choosing the more efficient outsourcing on the one hand, and the more strategic bankruptcy proof in-house production on the other. The other obvious benefit of in-house production is that it eliminates the problem of having to give incentives to an external supplier (in the form of share γ in our model). If the efficiency gains from eliminating this incentive problem are sufficiently large, in-house production would be preferred over outsourcing even in the absence of strategic bankruptcy caused by opportunistic behaviour by the supplier in case of outsourcing.

We now turn to the choice of outsourcing versus in-house production in the non-trivial case when outsourcing is not always preferred over in-house production or vice versa.

Proposition 6. *When*

a) $\rho_E = \rho_D$, *outsourcing is preferred over in-house production*

b) $\rho_E > \rho_D$ *there exists a threshold* $\overline{K}$ *such that the firm chooses in-house production when* $K \geq \overline{K}$ *and*

Row	$\rho_D < \rho_E$	$K < \overline{K}$	$\theta_H \leq \theta$	Production of y	Financing	Organisational outcome
1	✗	–	–	Outsource	Equity	Supplier-Equity holders
2	✓	✗	–	In-house	Debt	Equity-Debt holders
3	✓	✓	✗	Outsource	Mix	Supplier-Debt holders
4	✓	✓	✓	Outsource	Mix	All three parties

Table 1: Financing and organisational design outcomes

outsourcing when $K < \overline{K}$.

Proposition 6 characterises how the equity holders choose between outsourcing and the in-house production of y. The intuition for the result is the following. If the capital requirement is financed entirely with equity, the supplier cannot benefit by triggering bankruptcy and therefore equity financing eliminates strategic bankruptcy. It is therefore the preferred mode of financing when $\rho_E = \rho_D$, that is when equity is as cheap as debt. We see this in Row 1 of Table 1, which summarises the financing and organisational outcomes of the model. We see that the equity holders choose outsourcing and rely completely on equity when $\rho_E = \rho_D$.

When $\rho_E > \rho_D$, the equity holders face a trade-off. Although debt is attractive because it is cheaper, it creates the problem of strategic behaviour by the supplier which causes bankruptcy. This tension is resolved in one of two ways: If K is small enough, the equity holders choose outsourcing and typically finance it with a mix of equity and low levels of debt as seen in Rows 3-4 of Table 1. This is because the cost of using expensive equity is increasing in the level of capital requirement. In this case, if the realisation of θ is high enough (Row 4), the interests of all three stakeholders are aligned with maximising the firm value, and strategic bankruptcy is avoided in an outcome that we can think of one where the interests of all stakeholders aligned. If the realisation of θ is low enough (Row 3), the supplier triggers strategic bankruptcy.

If, on the other hand, the capital requirement K is large, then the attraction of cheap debt induces the equity holders to choose the less efficient in-house production as seen in Row 2 of Table 1. Although in-house production is less efficient, it solves the problem of opportunistic behaviour by the supplier.

5.1. Discussion of results

We now summarise the mechanics of our model. Ex-ante the equity holders choose $\gamma = \gamma^*$ as the supplier's share of revenue. Once θ is realised, there are two possibilities. If $\theta \geq D$ there are sufficient funds to pay the debt holders and the contract that is made ex-ante can be enforced. If however $\theta < D$, the supplier's effort is needed to ensure solvency and we assume that this shifts the bargaining power to the supplier, allowing him to pick $\gamma = 1$.

Here are some implications of the model:

1) The possibility of opportunistic behaviour by the supplier in the case of distress implies that the equilibrium capital structure is not the first best. In particular, there exists a range where $\rho_E > \rho_D$, and though debt is cheaper, equity financing is chosen to eliminate the possibility of strategic bankruptcy.

2) The first best is feasible only when $\rho_E = \rho_D$: that is when equity is as cheap as debt. When $\rho_E > \rho_D$, there is one of two inefficiencies:

   (a) The equilibrium mix of debt and equity is not efficient as the firm moves away from cheap debt towards costly equity to remove the debt holders so that the supplier cannot benefit through strategic bankruptcy, or

   (b) The equity holders substitute away from more efficient outsourcing to less efficient in-house production, in order to remove the (efficient) supplier from the picture.

3) The distribution of θ affects whether input y is produced in-house or outsourced. For instance, if $G(\theta_H) = 0$, there is no threat of strategic bankruptcy and the equity holders will choose outsourcing. As seen in Proposition 3, as the distribution of θ improves in the first order stochastic dominance sense, the likelihood of strategic default decreases, allowing the firm to increase the level of debt.

4) Our analysis suggests the following interpretation of bankruptcy: Bankruptcy is the only mechanism through which departure from the agreed upon allocations can be achieved. In particular, bankruptcy makes it possible for the supplier to increase his share of the payoff from the initially contracted upon $\gamma^* < 1$ to 1. Total payoff decreases as a result of the equity holders' zero investment in such case, but the supplier's payoff nonetheless increases. Note that the debt holders' payoff too decreases. We see this since $v_E(x(\gamma), y(\gamma); \theta)$ and $v_S(x(\gamma), y(\gamma); \theta)$ are both positive for $\theta_L \leq \theta \leq \theta_H$. This implies that the surplus in the case $x(\gamma)$ and $y(\gamma)$ are chosen, is large enough to permit the servicing of debt. Yet the debt holders' receive $\theta \leq \theta_H = D$. As such, the debt holders do not profit from bankruptcy.[18] Of course, the debt holders will not benefit from a regime where strategic bankruptcy is exogenously ruled out since their interest payment would adjust downwards to reflect the improvement in the probability of repayment.

5.2. Discussion of assumptions

Finally, before we conclude, it is worthwhile considering the role of the specific assumptions that are built into the set-up we have considered here.

The key assumption of this paper is that it is the supplier (in the case of outsourcing) that has an option to enter an agreement with the debt holders to eliminate the existing equity holders and to effectively become the new owner. This implies that there is no possibility of ex-post renegotiation between the supplier and

[18] However, in Appendix Appendix B we show that the debt holders receive a payoff greater than θ from bankruptcy when they are endowed with some bargaining power.

the equity holders. A deal where the equity holders provide input x creates room for renegotiations over the extra surplus that the two parties (equity holders and the suppliers) bring to the table. This would eliminate the possibility of strategic bankruptcy without affecting the region where non-strategic bankruptcy occurs.

Similarly, the paper also assumes no renegotiation between the equity and debt holders. In the case of in-house production, allowing for renegotiation between these two stakeholders will eliminate the inefficiency resulting from bankruptcy but will not change the inefficiency that results from the suboptimal production of input y in-house. Hence, introduction of renegotiation will not affect our results qualitatively in the case of in-house production, and will in fact make inefficient in-house production more attractive relative to efficient outsourcing. On the other hand, in case of outsourcing, renegotiation between the equity and debt holders will not affect our results at all due to the critical nature of input y in the production process – with outsourcing, the equity and debt holders cannot generate any surplus without the participation of the supplier.

One of our primary objectives is to explain the empirical phenomena of "debt conservatism" among the firms that are dependent on the supply of critical and non-substitutable inputs. In our set-up, this is achieved by endowing the supplier with some power in renegotiations as he can affect the size of the surplus by varying the supply of the specialised input. However, one may argue that equity holders could also participate in the ex-post alliance with the supplier and the debt holders. Effectively our model assumes that there are frictions when it comes to the participation of the equity holders in such an ex-post alliance. This assumption may be justified on the following additional grounds.

In our model, parties can negotiate with each other once, but not renegotiate once the agreement reached with earlier negotiations is vitiated by one of the parties.[19] In keeping with this, the equity holders are allowed to enter into an agreement with the supplier ($\gamma(\theta)$) and debt holders (D^*). However, once a node is reached where one of the parties has not complied with its commitment to this agreement, the resulting breakdown cannot be cured by renegotiation. Consequently, once the supplier reneges on the input he has agreed on, we assume that the equity holders cannot in good faith negotiate with him at the post-bankruptcy stage. This however does not affect possible negotiations between the supplier and debt holders as their relationship has not been subject to the breakdown of a previous agreement and hence the two can negotiate with a clean slate. Our model may therefore apply to situations where breakdown of an agreement, particularly due to opportunistic behaviour by one of the parties, creates frictions that impede renegotiations between the same parties.

In a general model, we may expect that there is a distribution of frictions from which nature picks the

[19] We thank an anonymous referee for suggesting this line of reasoning.

realisation. Realised frictions determine the cost of tripartite negotiations at the node where bankruptcy is triggered.[20] In the case of low frictions, the three parties would negotiate leading to the efficient outcome. The inefficiency associated with strategic bankruptcy would disappear. In this case, working backwards, the ex-ante choice between debt and equity would simplify to picking the cheaper option. On the other hand, in case of high frictions, tripartite negotiations will not be feasible. Our model can be viewed as a special case of such a general model when the distribution of frictions is such that tripartite negotiations are never feasible. As this distribution of frictions improves (leading to lower realised frictions), strategic bankruptcy becomes less likely and the model converges to a Modigliani-Miller world with parity between debt and equity financing when $\rho_D = \rho_E$. Of course if debt is cheaper than equity, that is $\rho_D < \rho_E$, then with sufficiently low frictions only debt financing will be used.

We have also made two other assumptions in the paper to meet our primary objective of establishing debt conservatism and the choice of firm boundaries. First, that debt is cheaper than equity, and second, that in-house production is costlier than outsourcing the production of the input y. The first assumption, discussed in Footnote 6, is made to ensure an interior solution in the optimal debt-equity ratio. In the absence of this, equity would always be preferred over debt. The second assumption, discussed in Footnote 8, is made to ensure the presence of a trade-off between outsourcing and in-house production. In the absence of this assumption, in-house production would always be preferred over outsourcing leading to the elimination of strategic bankruptcy.

6. Conclusion

We have addressed the issue of stakeholder behaviour during bankruptcy and the impact this has on the ex-ante choice of capital structure and the choice of outsourcing or the in-house production of a critical input. We have shown that both decisions are influenced by the risk and cost associated with a strategic bankruptcy that is triggered by the supplier. Due to friction in the contracting environment, namely the inability of the equity holders to control the behaviour of other stakeholders after bankruptcy, the equity holders choose a capital structure and firm boundaries to minimise the effect of ex-post strategic stakeholder behaviour. In particular, if the endogenous strategic bankruptcy costs are too high, one of two things happens:

1. The firm shuts out the supplier and produces the critical input in-house, even though the cost of in-house production is greater than outsourcing.
2. The firm shuts out the debt holders and funds the capital requirement through more expensive equity.

[20] There are other constraints on debt renegotiations discussed in the literature. The debt holders may commit ex-ante to not renegotiate the terms of the loan with the equity holders to prevent the latter from defaulting in a repeated relationship as in Hart and Moore (1998).

Since strategic bankruptcy is the central theme of our paper, it would be interesting to examine how bankruptcy laws in different countries affect strategic stakeholder behaviour. For example, a Chapter 11 restructuring in the United States allows for an automatic stay on payments of interests, emergency loan provision (debtor-in-possession financing), and a cram-down of reorganisation plans by judges on stakeholders. Our model predicts that such provisions may affect both the capital structure and the degree of vertical integration. It may be interesting to investigate this empirically in future work.

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Appendix A. Appendix

Proof of Lemma 1. Recall that

$$\begin{aligned} & v_S(x,y;\theta) := \gamma(R_S(y)+R_E(x)+\theta-D)-C_S(y), \\ \text{and} \quad & v_E(x,y;\theta) := (1-\gamma)(R_S(y)+R_E(x)+\theta-D)-C_E(x) \end{aligned} \tag{A.1}$$

where $x(\gamma)$ and $y(\gamma)$ are the solutions to the first-order conditions such that

$$(1-\gamma)R_E'(x(\gamma)) = C_E'(x(\gamma)) \qquad \text{and} \qquad \gamma R_S'(y(\gamma)) = C_S'(y(\gamma)). \tag{A.2}$$

Unique solutions $x(\gamma)$ and $y(\gamma)$ are guaranteed to exist by concavity of the revenue functions and convexity of the costs.

Define $\theta_H := D$ and $\theta_L := \max\{\theta\}$ such that

$$\begin{aligned} &\text{either} \quad v_S(x(\gamma),y(\gamma);\theta) \leq 0, \\ &\text{or} \quad v_S(0,y(\gamma);\theta) \leq 0 \qquad \text{and} \quad v_E(x(\gamma),y(\gamma);\theta) \leq 0 \end{aligned} \tag{A.3}$$

implying

$$\begin{aligned} \theta_L = \quad \max\{ & \tfrac{C_S(y(\gamma))}{\gamma} + D - R_S(y(\gamma)) - R_E(x(\gamma)), \\ & \min\{\tfrac{C_S(y(\gamma))}{\gamma} + D - R_S(y(\gamma)), \tfrac{C_E(x(\gamma))}{1-\gamma} + D - R_S(y(\gamma)) - R_E(x(\gamma))\}\} \end{aligned} \tag{A.4}$$

Note first that $\theta_L < \theta_H$. This is because

$$v_S(x(\gamma),y(\gamma);D) > 0 \qquad \text{and} \qquad v_E(x(\gamma),y(\gamma);D) > 0. \tag{A.5}$$

We can now analyse the three regions.

- Non-strategic bankruptcy when $\theta \leq \theta_L$.

  1. $v_S(x(\gamma),y(\gamma);\theta) \leq 0$. When $v_S(x(\gamma),y(\gamma);\theta) \leq 0$, the supplier's payoff from supplying positive input is non-positive. Hence withholding y is optimal. When the supplier withholds y the payoff of the equity holders is 0 (since $\theta_L < D$) and hence $x = 0$ is optimal. Since $\theta \leq \theta_L < \theta_H = D$, the profit of the firm is negative and it must go bankrupt.
  2. $v_E(x(\gamma),y(\gamma);\theta) \leq 0$ and $v_S(0,y(\gamma);\theta) \leq 0$. When $v_E(x(\gamma),y(\gamma);\theta) \leq 0$, the equity holders' payoff from supplying positive input is non-positive. Hence $x = 0$ is optimal. When the equity

holders supply $x = 0$ the payoff of the supplier is $v_S(0, y(\gamma); \theta) \leq 0$ and hence withholding y is optimal. Since $\theta \leq \theta_L < \theta_H = D$, the profit of the firm is negative and it must go bankrupt.

- Strategic bankruptcy when $\theta_L < \theta < \theta_H$. We define strategic bankruptcy as the state of bankruptcy that is triggered by a less than optimal choice of y by the supplier in Stage 3. This is only possible when $\theta < \theta_H$: since otherwise the debt holders are guaranteed their contractually obligated amount D.

  The supplier's payoff in Stage 4 at the end of post-bankruptcy negotiations with the debt holders is $R_S(y(1)) - C_S(y(1))$ from the expression in (3). On the other hand, the supplier's payoff from supplying $y(\gamma)$ as per the contract with the equity holders is

$$v_S(x(\gamma), y(\gamma); \theta) = \gamma(R_E(x(\gamma)) + R_S(y(\gamma)) + \theta - D) - C_S(y(\gamma)). \tag{A.6}$$

  At $\theta = \theta_H = D$, we see that $v_S(x(\gamma), y(\gamma); \theta_H) < R_S(y(1)) - C_S(y(1))$ by Assumption 1 since $v_S(x(\gamma), y(\gamma); \theta_H)$ is increasing in γ. Moreover, since $v_S(x(\gamma), y(\gamma); \theta)$ is increasing in θ, it follows that $R_S(y(1)) - C_S(y(1))$, the payoff from triggering bankruptcy, is greater than $v_S(x(\gamma), y(\gamma); \theta)$ for all $\theta < \theta_H$. Finally, since the supplier withholds y when $\theta < \theta_H$, it follows that $x = 0$ is optimal since the supplier's input is critical in the production function described in equation (1).

- Solvency when $\theta_H \leq \theta$. When $\theta_H \leq \theta$, we have solvency as the debt holders receive full payment D, the supplier receives $p(\theta) = v_S(x(\gamma), y(\gamma); \theta) > 0$, and the equity holders receive $e(\theta) = v_E(x(\gamma), y(\gamma); \theta) > 0$.

□

Proof of Proposition 1. Since (12) binds with equality, and substituting for w in (11) the problem reduces to choosing the function $\gamma(\theta)$ such that

$$\begin{aligned} \max_{\gamma(\theta)} \quad & \textstyle\int_{\theta_H}^{\infty} (R_S(y(\gamma(\theta))) + R_E(x(\gamma(\theta))) + \theta - D - C_S(y(\gamma(\theta))) - C_E(x(\gamma(\theta))))g(\theta)d\theta \\ & + G(\theta_H)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D). \end{aligned} \tag{A.7}$$

The first-order condition for this maximisation problem is

$$(R'_S(y(\gamma(\theta))) - C'_S(y\gamma(\theta)))y'(\gamma(\theta)) + (R'_E(x(\gamma(\theta))) - C'_E(x(\gamma(\theta))))x'(\gamma(\theta)) = 0. \tag{A.8}$$

Note that $R_S(y), C_S(y), R_E(x)$ and $C_E(x)$ are unaffected by the value of θ. Moreover, the optimal $x(\gamma)$ and $y(\gamma)$ defined in (A.2) are also independent of θ. Consequently, conditional on the supplier complying with the contract with the equity holders, the realisation of θ does not affect the choice of efforts x and y. This implies that $\gamma(\theta)$ must be a constant γ^*.

We know that the first-order condition for the choice of x by the equity holders and y for the supplier will hold. This implicitly defines $x(\gamma^*)$ and $y(\gamma^*)$ with

$$\gamma^* R_S'(y(\gamma^*)) = C_S'(y(\gamma^*)) \qquad \text{and} \qquad (1-\gamma^*)R_E'(x(\gamma^*)) = C_E'(x(\gamma^*)). \tag{A.9}$$

The second-order condition is

$$\begin{aligned} &(R_S''(y(\gamma)) - C_S''(y(\gamma)))y'(\gamma)^2 + (R_S'(y(\gamma)) - C_S'(y(\gamma)))y''(\gamma) \\ &+(R_E''(x(\gamma)) - C_E''(x(\gamma)))x'(\gamma)^2 + (R_E'(x(\gamma)) - C_E'(x(\gamma)))x''(\gamma) \quad < 0. \end{aligned} \tag{A.10}$$

We can check that this is satisfied: First, note that $R_S''(y) - C_S''(y)$ and $R_E''(x) - C_E''(x)$ are negative by the concavity of the revenue functions and the convexity of the cost functions. Next, note that $y'(\gamma)^2$ and $x'(\gamma)^2$ are always positive. This implies that the first and the third term in (A.10) must be negative. Next, note that $R_S'(y(\gamma)) - C_S'(y(\gamma))$ must be positive since $\gamma R_S'(y(\gamma)) = C_S'(y(\gamma))$, and similarly $R_E'(x(\gamma)) - C_E'(x(\gamma))$ since $(1-\gamma)R_E'(x(\gamma)) = C_E'(x(\gamma))$ by the first order condition in (A.2). For the final step we show that $x''(\gamma)$ and $y''(\gamma)$ are negative. Using the first-order conditions that define $x(\gamma), y(\gamma)$ in (A.2) we can derive $x'(\gamma)$, $y'(\gamma)$, $x''(\gamma)$ and $y''(\gamma)$. These are

$$x'(\gamma) = \frac{R_E'(x(\gamma))}{(1-\gamma)R_E''(x(\gamma)) - C_E''(x(\gamma))} < 0, \qquad y'(\gamma) = -\frac{R_S'(y(\gamma))}{\gamma R_S''(y(\gamma)) - C_S''(y(\gamma))} > 0, \tag{A.11}$$

$$x''(\gamma) = \frac{-2R_E''(x)x'(\gamma) + x'(\gamma)^2((1-\gamma)R_E'''(x) - C_E'''(x))}{C_E''(x) - (1-\gamma)R_E''(x)} < 0 \tag{A.12}$$

and

$$y''(\gamma) = \frac{2R_S''(y)y'(\gamma) + y'(\gamma)^2(\gamma R_S'''(y) - C_S'''(y))}{C_S''(y) - \gamma R_S''(y)} < 0. \tag{A.13}$$

$R_E'''(x) \leq 0, R_S'''(y) \leq 0, C_S'''(y) \geq 0$ and $C_E'''(x) \geq 0$ by the assumptions we have made about the production and the cost functions. Therefore the solution found for γ^* in (A.8) is the unique maximum.

Using the first-order conditions in (A.9) we can substitute for $C_E'(x(\gamma))$ and $C_S'(y(\gamma))$ in (A.8) and we get

$$\frac{1-\gamma^*}{\gamma^*} = -\frac{R_E'(x(\gamma^*)}{R_S'(y(\gamma^*))} \cdot \frac{x'(\gamma^*)}{y'(\gamma^*)}. \tag{A.14}$$

Note that $-\frac{x'(\gamma^*)}{y'(\gamma^*)} \geq 0$ since $x'(\gamma) \leq 0$. Holding $-\frac{x'(\gamma^*)}{y'(\gamma^*)}$ constant, note that the value of γ^* is increasing in $\frac{R_E'(x(\gamma^*)}{R_S'(y(\gamma^*))}$, which is the return from the equity holders' input relative to the supplier's input.

The assumption that $R_S(y)$ satisfies the Inada condition ensures that the first-order condition in (A.8) is greater than 0 at $\gamma = 0$. If the condition is always greater than 0, then the solution to γ^* is 1. $\square$

Proof of Proposition 2. Consider the case when $\frac{\rho_E}{\rho_D} = \rho > 1$. From (17), the equity holders's problem is to

$$\begin{aligned}\max_D \quad & \int_D^\infty \left(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) + \theta + (\rho - 1)D - C_E(x(\gamma^*)) - C_S(y(\gamma^*))\right) g(\theta)d\theta \\ & + \int_0^D \left(R_S(y(1)) - C_S(y(1)) + \rho\theta\right) g(\theta)d\theta - \rho_E K.\end{aligned} \tag{A.15}$$

This yields the following first-order condition

$$(\rho - 1)(1 - G(D^*)) - \underbrace{\left(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - R_S(y(1)) + C_S(y(1))\right)}_{\Delta TS} g(D^*) = 0. \tag{A.16}$$

The second-order condition for the problem to be concave is

$$-(\rho - 1)g(D^*) - g'(D^*)\Delta TS \leq 0. \tag{A.17}$$

We only consider the case when the objective function is concave. The objective function is concave when

$$g'(\theta) \geq 0 \qquad \text{or} \qquad \Delta TS \leq -\frac{(\rho - 1)g(\theta)}{g'(\theta)}. \tag{A.18}$$

With concavity, there are three possible sub cases to consider.

1. If $(\rho - 1)(1 - G(D)) - \Delta TSg(D) < 0$ for all D, then $k_D^* = 0$ and the entire capital K is financed through equity.
2. If $(\rho - 1)(1 - G(D)) - \Delta TSg(D) > 0$ for all D, then $k_D^* = K$ and the entire capital K is financed through debt.
3. If there exists a D that satisfies $(\rho - 1)(1 - G(D)) - \Delta TSg(D) = 0$, then there is a unique interior solution to D (by concavity) and $k_D^* \in (0, K)$.

Next consider the case when $\rho_E = \rho_D$ the equity holders' maximisation problem in (17) simplifies to choosing D to maximise

$$\begin{aligned}&(1 - G(D))\left(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_S(y(\gamma^*)) - C_E(x(\gamma^*))\right) \\ &+ G(D)\left(R_S(y(1)) - C_S(y(1))\right) + \int_0^\infty \theta g(\theta)d\theta - \rho_E K.\end{aligned} \tag{A.19}$$

This is a linear programming problem in D. Note that $y(1) = y(\gamma = 1)$. Since $R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_S(y(\gamma^*)) - C_E(x(\gamma^*)) > R_S(y(1)) - C_S(y(1))$ by the optimality of $\gamma = \gamma^*$, the optimal $D^* = 0$ implying

$k_D^* = 0$ by (16) and all of capital K is fully financed by equity. $\square$

Proof of Proposition 3. Defining $\frac{\rho_E}{\rho_D} =: \rho$, the first-order condition in (19) can be rearranged to

$$\frac{g(D^*)}{1-G(D^*)} = \frac{\rho - 1}{\Delta TS} \tag{A.20}$$

where the right hand side is independent of the distribution of θ.

We will first prove part a) of the proposition. To begin with, note that $F(\cdot)$ and $G(\cdot)$ must individually satisfy the MHRC since (A.17) the second-order condition for D^* that ensures concavity is

$$-(\rho-1)g(D^*) - g'(D^*)\Delta TS \leq 0$$

Since the second-order condition is satisfied by assumption, we find that $\frac{g(D^*)}{1-G(D^*)}$ and $\frac{f(D^*)}{1-F(D^*)}$ are both increasing in D^*. Moreover by the first-order condition we have

$$\frac{f(D^*(F(\cdot)))}{1-F(D^*(F(\cdot)))} = \frac{g(D^*(G(\cdot)))}{1-G(D^*(G(\cdot)))} = \frac{\rho-1}{\Delta TS}, \tag{A.21}$$

and consequently $D^*(F(\theta)) \geq D^*(G(\theta))$ because $F(\cdot)$ dominates $G(\cdot)$ in the MHRC sense by assumption, that is

$$\frac{f(D^*)}{1-F(D^*)} \leq \frac{g(D^*)}{1-G(D^*)} \qquad \forall D^*. \tag{A.22}$$

We now turn to part b) note that

$$\begin{aligned}
\tfrac{\partial \Delta TS}{\partial c_S} &= \underbrace{((R_S'(y(\gamma^*)) - c_S))y'(\gamma^*) + (R_E'(x(\gamma^*)) - c_E)x'(\gamma^*))}_{0 \text{ by FOC for } \gamma \text{ in (A.8)}} \tfrac{\partial \gamma}{\partial c_S} - y(\gamma) \\
&\quad - \underbrace{(R_S'(y(1)) - c_S)}_{0 \text{ by FOC for } y(1) \text{ in (2)}} \cdot \tfrac{\partial y(1)}{\partial c_S} + y(1) \qquad\qquad \text{(A.23)} \\
&= y(1) - y(\gamma) \geq 0,
\end{aligned}$$

since $y(1) = y(\gamma = 1) \geq y(\gamma)$. Similarly

$$\frac{\partial \Delta TS}{\partial c_E} = \underbrace{((R_S'(y(\gamma^*)) - c_S))y'(\gamma^*) + (R_E'(x(\gamma^*)) - c_E)x'(\gamma^*))}_{0 \text{ by FOC for } \gamma \text{ in (A.8)}} \frac{\partial \gamma}{\partial c_E} - x(\gamma) \leq 0. \tag{A.24}$$

Hence differentiating the FOC for D^* in (19) and defining $\rho := \frac{\rho_E}{\rho_D}$ we find that

$$\frac{\partial D^*}{\partial c_S} = \frac{g(D^*)\frac{\partial \Delta TS}{\partial c_S}}{(\rho-1)g(D^*) + g'(D^*)\Delta TS} \geq 0 \tag{A.25}$$

and

$$\frac{\partial D^*}{\partial c_E} = \frac{g(D^*)\frac{\partial \Delta TS}{\partial c_E}}{(\rho-1)g(D^*) + g'(D^*)\Delta TS} \leq 0. \tag{A.26}$$

Note that the denominator in (A.25) and (A.26) is positive by the second-order condition for D^* derived in (A.17). □

Proof of Lemma 2. Recall that x_i, y_i are the values of x and y that solve the first-order conditions

$$R'_E(x_i) = \phi C'_E(x_i) \qquad \text{and} \qquad R'_S(y_i) = \phi C'_S(y_i). \tag{A.27}$$

Define

$$\theta_B := \max\{0, D + \phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i)\}. \tag{A.28}$$

By inspection, we see that if $\theta < \theta_B$, the payoff of the equity holders from exerting any positive input is negative. Hence $x = y = 0$ is optimal. In this case $D > \theta$ and consequently the firm must go bankrupt.

Similarly, when $\theta_B \leq \theta$, we see that the payoff of the equity holders from supplying x_i, y_i that solve equations in (A.27) is non-negative. In this case the debt holders will be paid in full and the equity holders will claim what remains and the firm will be solvent. □

Proof of Proposition 4. Define $\rho := \frac{\rho_E}{\rho_D}$. First consider the case when $\rho > 1$. From (34), the equity holders' problem is to

$$\begin{aligned}
&\max_D && \textstyle\int_{\theta_B(D)}^{\infty} \left(R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i)\right) g(\theta)d\theta \\
& && \textstyle +\rho \int_0^{\theta_B(D)} \theta g(\theta)d\theta + \rho(1 - G(\theta_B(D)))D - \rho_E K \\
\Leftrightarrow\ &\max_D && \textstyle (1 - G(\theta_B(D)))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) + \int_0^{\infty} \theta g(\theta)d\theta \\
& && \textstyle +(\rho-1)\int_0^{\theta_B(D)} \theta g(\theta)d\theta + (\rho-1)(1 - G(\theta_B(D)))D - \rho_E K.
\end{aligned} \tag{A.29}$$

Since $\theta_B = D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)$ and $\frac{\partial \theta_B}{\partial D} = 1$, the maximisation problem above yields the following first-order condition when $\theta_B > 0$

$$(\rho-1)(1 - G(\theta_B(D_i^*))) - \rho\left(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)\right) g(\theta_B(D_i^*)) = 0, \tag{A.30}$$

where D_i^* is the optimal payment to the debt holders in case of solvency when in-house (i) production is used. The second-order condition for the problem to be concave is

$$-(\rho-1)g(\theta_B(D_i^*)) - \rho g'(\theta_B(D_i^*))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) \leq 0. \tag{A.31}$$

We only consider the case when the objective function is concave. The objective function is concave when

$$g'(\theta) \geq 0 \qquad \text{or} \qquad R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) \leq -\frac{(\rho-1)g(\theta)}{\rho g'(\theta)}. \tag{A.32}$$

Recall that $\theta_B = \max\{0, D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)\}$. There are two cases to consider

1. $\rho_D K - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i) \leq 0$. In this case the firm is always solvent since the principal and the interest $\rho_D K$ can be paid back even with $\theta = 0$. Consequently, $\theta_B(D) = 0$ and capital requirement K is financed entirely through debt. Hence, $k_D^* = K$ and $D^* = \rho_D K$.
2. $\rho_D K - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i) > 0$. An interior solution for k_D^* is now possible. The first-order condition in equation (A.30) can be rewritten as

$$\left(1 - \frac{1}{\rho}\right)(1 - G(\theta_B(D_i^*))) - (R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))\, g(\theta_B(D_i^*)) = 0. \tag{A.33}$$

   There are 3 sub cases to consider:

   (a) If there exists a D that (A.33) holds, then there is a unique interior solution to D (by concavity) and $k_D^* \in (0, K)$.

   (b) If the left hand side of (A.33) is greater than 0 for all D, then $k_D^* = K$ and the entire capital requirement K is financed through debt.

   (c) If the left hand side of (A.33) is less than 0 for all D, then $k_D^* = 0$ and the entire capital requirement K is financed through equity.

Next, consider the case when $\rho_E = \rho_D$, that is $\rho = 1$. The equity holders' maximisation problem simplifies to choosing D to maximise

$$(1 - G(\theta_B(D)))\,(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))) + \textstyle\int_0^\infty \theta g(\theta) d\theta - \rho_E K. \tag{A.34}$$

Recall that $\theta_B(D) = \max\{0, D + \phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i)\}$. The maximum is attained when $\theta_B(D) = 0$. This is possible when $k_D^* = 0$ by (33) and the entire capital requirement K is financed by equity.

□

Proof of Proposition 5. Recall that $\rho := \frac{\rho_E}{\rho_D}$. As shown in Propositions 2 and 4, when $\rho \leq 1$ we have $k_D^* = 0$ for both in-house production and outsourcing. As we increase ρ we need to assume that the inequalities in (A.18) and (A.32) that ensure concavity continue to hold. In this case there is a threshold ρ below which $k_D = 0$ and above which there is an interior solution to k_D defined by the first-order condition in (A.16) for outsourcing and (A.30) for in-house production.

- First, consider outsourcing. In this case we derive the comparative statics of D^* with respect to ρ using (A.16) and find

$$\frac{\partial D^*}{\partial \rho} = \frac{1 - G(D^*)}{\Delta TS g'(\theta) + (\rho - 1) g(D^*)} > 0, \tag{A.35}$$

where $\Delta TS = R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - R_S(y(1)) + C_S(y(1))$. This implies that in this region $\frac{\partial k_D^*}{\partial \rho} > 0$.

- Next, consider in-house production. In this case, we derive the comparative statics of D_i^* with respect to ρ using (A.30) and find

$$\frac{\partial D_i^*}{\partial \rho} = \frac{1 - G(\theta_B(D_i^*)) - g(\theta_B(D_i^*))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))}{(\rho - 1) g(\theta_B(D_i^*)) + \rho(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) g'(\theta_B(D_i^*))} > 0. \tag{A.36}$$

The numerator is positive due to equation (A.30) and the denominator is positive by the concavity condition in inequality (A.32). Recall that

$$\theta_B = \max\{0, D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)\} \tag{A.37}$$

and

$$\rho_D k_D^* = \int_0^{\theta_B} \theta g(\theta) d\theta + (1 - G(\theta_B)) D \tag{A.38}$$

Totally differentiating (A.38) with respect to ρ while implicitly defining $\rho_D = \frac{\rho_E}{\rho}$ we find

$$\rho_D \frac{\partial k_D^*}{\partial \rho} = (1 - G(\theta_B) - g(\theta_B)(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))) \frac{\partial D^*}{\partial \rho} + k_D^* \frac{\rho_E}{\rho^2} > 0. \tag{A.39}$$

This is true since we have seen that $\frac{\partial D^*}{\partial \rho} > 0$ and $1 - G(\theta_B) > g(\theta_B)(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))$ by (A.30).

Finally, as we increase ρ, there may be a threshold of ρ such that the respective first-order conditions hold for in-house production and outsourcing, such that $k_D^* = K$. For ρ greater than this, k_D^* is invariant to increase in ρ. This implies that k_D is weakly increasing in ρ when the objective function is concave.

□

Proof of Proposition 6. As discussed, the interesting parameter space is when

$$\begin{aligned} & R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) \\ \geq \; & R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) \\ \geq \; & R_S(y(1)) - C_S(y(1)). \end{aligned} \tag{A.40}$$

Outside of this region, when $R_E(x_i)+R_S(y_i)-\phi C_E(x_i)-\phi C_S(y_i)$ is larger than the first term (smaller than the last term), the equity holders always prefer in-house production (outsourcing).

When Assumption 2 is satisfied, there is no bankruptcy with in-house production. Capital requirement K is financed entirely through debt since $\rho_E \geq \rho_D$, and the payoff of the equity holders is

$$\pi_E^i = R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + \mathbb{E}(\theta) - \rho_D K. \tag{A.41}$$

On the other hand, with outsourcing, the payoff of the equity holders is

$$\begin{aligned}
\pi_E^o = \quad & (1-G(D^*))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) \\
& +G(D^*)(R_S(y(1)) - C_S(y(1))) + \mathbb{E}(\theta) - \textstyle\int_0^{D^*} \theta g(\theta) d\theta - D^*(1-G(D^*)) \\
& -\rho_E(K-k_D^*) \\
= \quad & (1-G(D^*))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) \\
& +G(D^*)(R_S(y(1)) - C_S(y(1))) + \mathbb{E}(\theta) - \rho_D k_D^* - \rho_E(K-k_D^*),
\end{aligned} \tag{A.42}$$

since $\rho_D k_D^* = \int_0^{D^*} \theta g(\theta) d\theta + D^*(1-G(D^*))$.

Appendix A.0.0.1. Consider case (a) where $\rho_E = \rho_D$.. In this case, K will be financed through equity and outsourcing will be preferred since $k_D^* = D^* = 0$ and $\pi_E^o > \pi_E^i$ simplifies to

$$\begin{aligned}
& R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) + \mathbb{E}(\theta) - \rho_E K \\
> \quad & R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + \mathbb{E}(\theta) - \rho_D K,
\end{aligned} \tag{A.43}$$

which is true because of the inequalities in (A.40).

Appendix A.0.0.2. Consider case (b) where $\rho_E > \rho_D$.. Capital continues to be financed entirely through debt for the case of in-house production. There are three possibilities for capital financing with outsourcing.

1. First, consider the case when capital is financed entirely through equity with outsourcing and $k_D^* = 0$. In this case $D^* = 0$ and $\pi_E^o > \pi_E^i$ simplifies to

$$\begin{aligned}
& R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) \\
> \quad & R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + (\rho_E - \rho_D)K.
\end{aligned} \tag{A.44}$$

   Since the last term is increasing in K, there exists a threshold $\overline{K}_1$ such that in-house production with debt financing is preferred.

2. Second, consider the case when capital is financed entirely through debt with outsourcing. In this case, $k_D^* = K$ and $\pi_E^o > \pi_E^i$ simplifies to

$$\begin{aligned}
& (1-G(D^*))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) \\
& +G(D^*)(R_S(y(1)) - C_S(y(1))) \\
> \;\; & R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i).
\end{aligned} \tag{A.45}$$

Since $\rho_D K = \int_0^{D^*} \theta g(\theta) d\theta + D^*(1-G(D^*))$, we see that D^* is increasing in K. Due to the inequalities in (A.40), there must exist a threshold for $\overline{K}_2$ such that (A.45) cannot hold.

3. Finally, consider the case when there is an interior solution to k_D^* and capital is financed through a mix of debt and equity with outsourcing. Note that when k_D^* is interior, there is no dependence of k_D^* on K. To see this, note that K only appears as a constant in the maximisation problem when the equity holders choose D^*, and that k_D^* is fully determined once the equity holders pick D^* as the optimal γ, x and y are fully anticipated. Consequently, $\pi_E^o > \pi_E^i$ simplifies to

$$\begin{aligned}
& (1-G(D^*))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) \\
& +G(D^*)(R_S(y(1)) - C_S(y(1))) \\
> \;\; & R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + (\rho_E - \rho_D)(K - k_D^*).
\end{aligned} \tag{A.46}$$

Since k_D^* and D^* remain constant, an increase in K leads to in-house production with only debt financing being preferred over outsourcing beyond some threshold level of $\overline{K}_3$.

Since the condition under which $\pi_E^o > \pi_E^i$ differs across the three cases, the thresholds of $\overline{K}$ in the three cases will be different.

□

Appendix B. Extension: Nash bargaining between the supplier and debt holders

In our model, with outsourcing in Stage 4, the supplier bargains with the debt holders. In this case we have assumed that the bargaining power is entirely with the supplier who consequently makes a take-it-or-leave-it offer to the debt holders. In this extension, we present a sketch of what would happen instead if we allowed the debt holders some bargaining power.

To begin with, note that the outside option of the debt holders, when the supplier withholds y, is θ and the additional surplus generated when y is supplied is $R_S(y) - C_S(y)$. Allowing for both to have some bargaining power, the payoffs of the supplier and the debt holders are $\beta(R_S(y) - C_S(y))$ and $\theta + (1-\beta)(R_S(y) - C_S(y))$, respectively.

Maximising his payoff, the supplier will choose $y(1)$ and the equilibrium payoffs of the supplier and the debt holders are $\beta(R_S(y(1)) - C_S(y(1)))$ and $\theta + (1-\beta)(R_S(y(1)) - C_S(y(1)))$. We now show that the existence of an interval in θ with strategic bankruptcy survives this modification. To see this, note that the supplier now triggers bankruptcy when

$$\gamma(R_E(x(\gamma)) + R_S(y(\gamma)) + \theta - D) - C_S(y(\gamma)) \leq \beta(R_S(y(1)) - C_S(y(1))). \tag{B.1}$$

This is because the left hand side is the supplier's payoff when he complies with the contract, whereas the right hand side is his payoff when he triggers bankruptcy and works with the debt holders. Given Assumption 1, this condition holds when $\beta = 1$ at $\theta = \theta_H = D$. Note that the left hand of (B.1) is continuous and increasing in θ. The payoff for the supplier from complying with his contract with the equity holders and exerting $y(\gamma)$ is non positive at $\theta \leq \theta_L$. On the other hand, the payoff from triggering strategic bankruptcy is strictly positive at $\beta(R_S(y) - C_S(y))$. Consequently, there must exist a threshold $\theta_H(\beta) \in (\theta_L, D)$, such that the supplier strictly prefers to trigger strategic bankruptcy when $\theta \in (\theta_L, \theta_H(\beta))$ to supplying $y(\gamma)$. In particular, $\theta_H(\beta)$ is simply the value of θ, such that condition (B.1) holds with equality.

By inspecting (B.1), we observe that the value of $\theta_H(\beta)$ increases in β. Hence, the strategic bankruptcy interval $(\theta_L, \theta_H(\beta))$ is increasing in β. Moreover, at $\beta = 0$ we will have $\theta_H(0) = \theta_L$ and the region with strategic bankruptcy will disappear entirely.

With this modification, we observe that the debt holders may be better off with strategic bankruptcy when $\beta \in (0,1)$ and $\theta \in (\theta_L, \theta_H)$. It is clear that the supplier prefers strategic bankruptcy to complying with his contract with the equity holders and supplying $y(\gamma)$. To see that the debt holders may be better off, note that in this region the payoff of the debt holders with strategic bankruptcy is $\theta + (1-\beta)(R_S(y(1)) - C_S(y(1)))$ whereas their payoff in the case of solvency is D. Since $\theta + (1-\beta)(R_S(y(1)) - C_S(y(1))) > D$ is not feasible even with $\theta < D$, it is possible that the debt holders profit from strategic bankruptcy. organisational As shown above, the region with strategic bankruptcy survives even when we allow the debt holders to have bargaining power at the post-bankruptcy stage when they negotiate with the supplier. This modification however makes the rest of the model less tractable since $\theta_H(\beta)$ now also depends on γ through its dependence on $x(\gamma)$ and $y(\gamma)$ as seen in condition (B.1). This complicates the equity holders' problem in Stage 2 when they choose the function $\gamma(\theta)$. This is because the second order condition in the optimisation problem is no longer straightforward. To avoid this complication, we continue to use the special case of $\beta = 1$ in our model.